Teacher notes
Topic B

Cooling curve for a radiating sphere

Consider two metallic spheres of the same material initially at the same temperature that radiate into space as black bodies. The spheres have different radii. Which one will it cool down to 0 K fastest?

The power radiated is $P = \sigma 4\pi R^2 T^4$. Then

$$mc\frac{\Delta T}{\Delta t} = -\sigma 4\pi R^2 T^4$$

This becomes

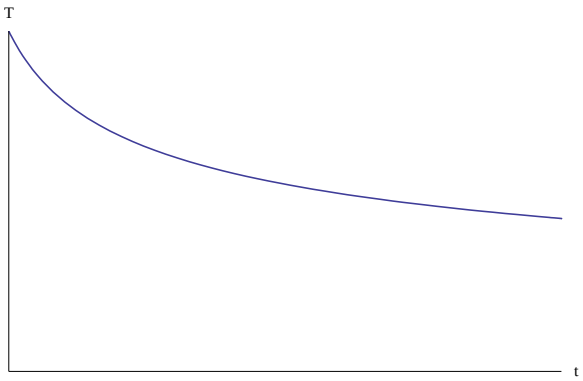
$$\rho\frac{4\pi}{3}R^3 c\frac{\Delta T}{\Delta t} = -\sigma 4\pi R^2 T^4$$
$$\frac{\Delta T}{\Delta t} = -\frac{3\sigma T^4}{c\rho R}$$

This shows that the rate of change of temperature of the small sphere is larger in magnitude.

Solving the differential equation $\frac{dT}{dt} = -\frac{3\sigma T^4}{c\rho R}$ gives

$$\frac{dT}{T^4} = -\frac{3\sigma}{c\rho R}dt$$
$$-\frac{1}{3T^3} = -\frac{3\sigma}{c\rho R}t - \frac{1}{3T_0^3}$$
$$\frac{1}{T^3} = \frac{9\sigma}{c\rho R}t + \frac{1}{T_0^3}$$
$$T = T_0\sqrt[3]{\frac{c\rho R}{9\sigma T_0^3 t + c\rho R}}$$

A graph of temperature versus time is shown according to this model.



A student uses this model to calculate the age of the Earth by supposing that at the time of its creation the Earth had a uniform temperature of T = 5500 K and a present surface temperature of 300 K.

Solving the last equation for time we find

$$t = \frac{c\rho R}{9\sigma}\left(\frac{1}{T^3} - \frac{1}{T_0^3}\right)$$

The following data are available:

R = 6400 km, $\rho = 5.5\times10^3$ kg m^{-3} and $c = 2\times10^3$ J kg^{-1} K^{-1}.

Calculate the age of the Earth based on this model. (About 162 000 years)

The estimate is clearly incorrect. Suggest reasons why this model is inappropriate. (The model assumes a uniform temperature throughout the volume of the sphere.)

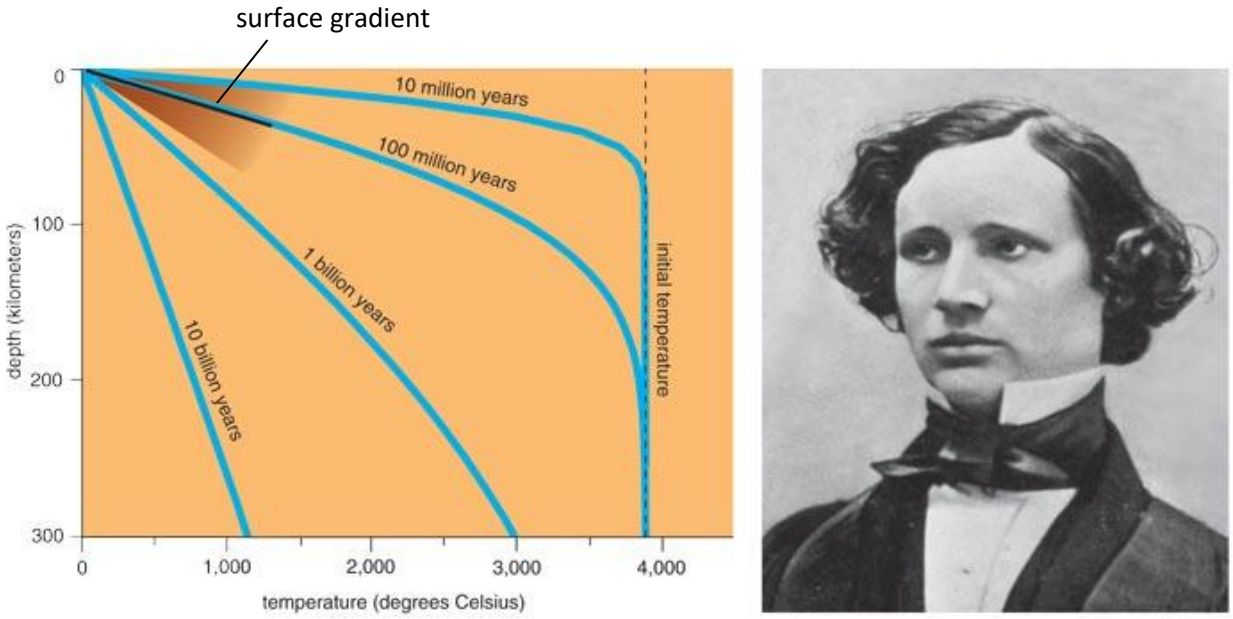
Lord Kelvin estimated the age of the Earth in 1844 and later in 1862. His estimate was an Earth age between 100 and 200 million years; his revised later estimate gave an age between 20 and 40 million years. These estimates are also wrong. The controversy around this estimate is a fascinating story about the scientific method, the fear of authority and how a wrong result led to new discoveries.

1. Find out a bit more about Kelvin's estimate.
2. What went wrong?
3. What impact did his work have on geology?
4. What impact did his work have on the theory of evolution?
5. Why is Kelvin sometimes championed by creationists?

6. The modern estimate of the age of the Earth is about 4.5 billion years. How was this estimate reached?

Key points

1. Kelvin assumed that at the time of its creation the Earth had a high, uniform temperature. As time went by, heat moving through the Earth was dissipated into space so that the surface reached a lower temperature. The temperature at a point in the interior of the Earth depended on both the time elapsed since creation as well as on the depth. Kelvin was able to show that the age of the Earth was related to the rate at which the temperature drops at the surface. His estimate of the age of the Earth was based on measurements of this surface temperature gradient.



From P. England, P. Molnar and F. Richter in *American Scientist*, July-August 2007, Vol. 95, No. 4, p. 342. The gradient at the surface depends on the age of the Earth. A young William Thomson (Lord Kelvin).

2. John Perry, Kelvin's own assistant, pointed out that in a molten Earth interior, convection would play a crucial role in transferring heat to the surface. His model assumed a solid Earth of depth 50 km at the surface and the rest a molten hot interior. Unlike Kelvin's model, there would now be a different rate for transferring heat in the molten and the solid parts: a high rate in the molten interior due to convection and a much lower rate in the solid part near the surface. The measured surface temperature gradient would then be consistent with an Earth age in the billions of years.

Another argument put forward in explaining where Kelvin went wrong was Kelvin's assumption that no additional source of energy was available inside the Earth other than the internal energy

of the Earth at the time of its creation. We now know that radioactivity of materials inside the Earth provides an additional source of energy. (Radioactivity was discovered in 1896; Kelvin started his calculations in 1840, at the age of 16 (!), and completed them 4 years later.) Many people still believe that this is the main error in Kelvin's estimate. Rutherford himself made this point in 1904 in a lecture at the Royal Institution. Rutherford said:

> I came into the room, which was half dark, and presently spotted Lord Kelvin in the audience and realized that I was in for trouble at the last part of the speech dealing with the age of the earth, where my views conflicted with his. To my relief he fell fast asleep but as I came to the important point, I saw the old bird sit up, open an eye and cock a baleful glance at me! Then sudden inspiration came, and I said Lord Kelvin had limited the age of the earth, *provided no new source of heat was discovered*. That prophetic utterance refers to what we are now considering tonight, radium! Behold! The old boy beamed at me.

However, detailed calculations show that this would not have changed Kelvin's calculations significantly; ignoring convection was the main error in Kelvin's estimate.

3. At the time of Kelvin geologists believed that the Earth was of "unlimited" age. Kelvin's work forced geologists to pay attention to the laws of Physics in describing geological phenomena.
4. Kelvin's low estimate of the Earth age came into conflict with evolutionary biologists who demanded a much older Earth. For this reason Kelvin was considered an anti-evolutionist as well as a reactionary.
5. Creationists, unlike proponents of evolution, demanded a low estimate of the Earth age. Such an estimate was provided by Kelvin's calculations.
6. By radioactive rock dating.